

Merger rates of black hole binaries: prospects for gravitational wave detectors

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Abstract. Mergers of black-hole binaries are expected to release large amounts of energy in the form of gravitational radiation. However, binary evolution models predict merger rates too low to be of observational interest. In this paper we explore the possibility that black holes become members of close binaries via dynamical interactions with other stars in dense stellar systems. In star clusters, black holes become the most massive objects within a few tens of millions of years; dynamical relaxation then causes them to sink to the cluster core, where they form binaries. These black-hole binaries become more tightly bound by superelastic encounters with other cluster members, and are ultimately ejected from the cluster. The majority of escaping black-hole binaries have orbital periods short enough and eccentricities high enough that the emission of gravitational waves causes them to coalesce within a few billion years. We predict a black-hole merger rate of 10^{-8} to 10^{-7} per year per cubic megaparsec, implying gravity-wave detection rates substantially greater than the corresponding rates from neutron star mergers. For the first generation Laser Interferometer Gravitational-Wave Observatory (LIGO-I), we expect about one detection during the first two years of operation. For its successor LIGO-II, the rate rises to roughly one detection per day. There is about an order of magnitude uncertainty in these numbers.

1 Introduction

Globular clusters contain about one hundred times more low-mass X-ray binaries (LMXBs) per unit mass than does the Galaxy as a whole—the Galaxy, with a mass of $2 \times 10^{11} M_{\odot}$, contains about 100 LMXBs, whereas the Galactic globular cluster population, with a total mass of just $2 \times 10^8 M_{\odot}$, contains at least 10. All known cluster LMXBs have neutron stars as primaries.

One might seek an explanation for this discrepancy in LMXB numbers in the obvious population differences between globular clusters and the Galactic disc. The disc contains a mixture of stellar populations, with broad ranges

in age and metallicity, while all stars in a given globular cluster have essentially the same age and initial composition. Conceivably, a globular cluster might experience a characteristic “LMXB-rich” epoch as its component stars evolved. This hypothesis, however, is not widely accepted.

A more likely explanation for the excess of LMXBs in globular clusters lies in the radically different dynamics of cluster stars compared to stars in the Galactic disc. The mean stellar density in the disc is about 0.1 star per cubic parsec, with relatively little variation from place to place. Globular clusters, on the other hand, exhibit a huge spread in densities, ranging from values close to the density in the disc near the cluster tidal radius, to tens of millions of stars per cubic parsec in the densest cluster cores. These density differences may be responsible for the higher birthrate of LMXBs in globular clusters relative to the Galactic disc—dynamical interactions favor the formation of LMXBs.

For a cluster age of ~ 10 Gyr, neutron stars are more than twice as massive as other cluster members. Dynamical friction causes them to sink to the center of the cluster potential well, where stellar densities are higher and encounters are much more common. Once in the core, close encounters with other stars may lead to two-body tidal capture (Fabian et al. 1975) or to three-body exchange interactions (Phinney & Sigurdsson 1991). In either case, the neutron star gains a low-mass companion, which later evolves to become the donor in an LMXB. High kick velocities imparted to newborn neutron stars cause the majority to be ejected from their parent clusters upon formation (Davies & Hansen 1998). Only about 20% of neutron stars are retained by globular clusters, yet cluster LMXBs still greatly outnumber the population in the Galactic disc. Mass segregation and tidal capture or exchange are evidently very efficient processes.

Given this reasoning, it is all the more striking that no black-hole X-ray binaries are observed in globular clusters. Black holes do not receive a kick upon formation in a supernova (White & van Paradijs 1996), so hardly any escape promptly. Black holes are also considerably more massive than neutron stars, causing them to sink in the cluster core even more rapidly. In equipartition, the black holes’ velocity dispersion is $v \propto m^{-1/2}$. Thus, the cross section for a dynamical interaction, which is dominated by gravitational focusing, is

$$\sigma \propto \frac{m}{\sqrt{v}} \propto m^{5/4}. \quad (1)$$

Hence, for a black hole mass of $10 M_{\odot}$, we would naively expect that globular clusters should contain almost an order of magnitude more LMXBs with black holes than with neutron stars. However, none are found. The explanation for this discrepancy is as follows.

2 Black hole formation

The initial mass function of globular clusters is well described by a Scalo (1986) distribution, with lower and upper limits of $0.1 M_{\odot}$ and $100 M_{\odot}$. This IMF has a mean mass $\langle m \rangle \sim 0.5 M_{\odot}$ and leads to the formation of about 5×10^{-4} black holes per star. A $10^6 M_{\odot}$ star cluster thus produces about 1000 black holes. Black holes resulting from stellar evolution are generally quite massive objects: known black hole masses range from 6 to $10 M_{\odot}$. For clarity we adopt a black hole mass of $m_{\text{bh}} = 10 M_{\odot}$; the precise value is not crucial to our discussion, so long as it significantly exceeds $\langle m \rangle$.

As with neutron stars, dynamical friction causes the black holes to sink to the cluster core. The mass segregation time scale is $\sim \langle m \rangle / m_{\text{bh}}$ half-mass relaxation times, or about 10^8 yr for $\langle m \rangle = 0.5 M_{\odot}$ and a cluster relaxation time of 10^9 years (see Kulkarni et al. 1992 and Sigurdsson & Hernquist 1992 for details). As mass segregation proceeds and the cluster core contracts, binaries are formed, providing the energy needed to support the core against further gravothermal collapse (Heggie 1975). The black holes will preferentially form binaries with one another, both because it is energetically favorable for them to do so, and because of the generally larger black-hole interaction cross sections. Subsequently, the black-hole binaries evolve via dynamical encounters with other cluster components. On average, each encounter between a black-hole binary and a single black hole hardens the binary (increases its binding energy) by about 20%. Two-thirds of the energy released goes into binary recoil, the rest into recoil of the other black hole involved in the interaction. The hardening process continues until the recoil velocity exceeds the clusters' escape speed and the black-hole binary is ejected from the cluster.

A binary can release enough energy to eject itself from the cluster once its binding energy exceeds ~ 1000 times the mean kinetic energy of cluster stars. By this time the binary has typically experienced some 40–50 hard encounters. The recoil energetics imply that, on average, a black-hole binary is ejected after its *previous* encounter has already ejected a single black hole. Thus, for each ejected black-hole binary one expects two single black holes to be ejected. There are two possible dynamical scenarios for binary ejection: (1) there will be at most one or two black-hole binaries in the core at any given time, and a new binary can form only after these are ejected; or (2) the core is able to support a large population of black-hole binaries. In the former case, the ejection process takes considerably longer, as the binaries are ejected sequentially. In the latter, the binaries may be ejected more or less simultaneously. Our simulations are not sufficiently detailed to discriminate between these alternatives.

In order to eject a black-hole binary following an encounter with a low-mass cluster member, the binding energy of the black-hole binary must exceed $\sim 4 \times 10^4 kT$. However, by this time, the black-hole binary has shrunk to such a small orbital separation that it likely merges due to emission of gravitational wave radiation before another encounter takes place. On the other hand, the

black-hole binary easily ejects low mass stars. The black hole binary starts to eject low mass stars as soon as its binding energy exceeds $\sim 25 kT$. At least 20 low mass stars are ejected for each single black hole.

3 Characteristics of ejected binaries

The energy of an ejected binary and its orbital separation are coupled to the dynamical characteristics of the star cluster. For a cluster in virial equilibrium, we have

$$kT = \frac{2E_{\text{kin}}}{3N} = \frac{-E_{\text{pot}}}{3N} = \frac{GM^2}{6Nr_{\text{vir}}}, \quad (2)$$

where M and N are the total cluster mass and number of stars, respectively, and r_{vir} is the virial radius. A black-hole binary with semi-major axis a has

$$E_b = \frac{Gm_{\text{bh}}^2}{2a}, \quad (3)$$

and therefore

$$\frac{E_b}{kT} = 3N \left(\frac{m_{\text{bh}}}{M} \right)^2 \frac{r_{\text{vir}}}{a}. \quad (4)$$

We can thus compute the properties of black-hole binaries produced by globular clusters of given masses and virial radii. These cluster parameters are assumed to be distributed as independent Gaussians with means and dispersions of $\log_{10} M = 5.5 \pm 0.5$ and $\log r_{\text{vir}} = 0.5 \pm 0.3$, respectively (Djorgovski & Meylan 1994). A recent parameter-space survey of cluster initial conditions (Takahashi & Portegies Zwart 2000) finds that typical globular clusters which have survived for a Hubble time have lost $\gtrsim 60\%$ of their initial mass and have expanded by about a factor of three. We correct for this by changing the adopted distributions to $\log_{10} M = 6.0 \pm 0.5$ and $\log r_{\text{vir}} = 0 \pm 0.3$.

4 Production of gravitational radiation

An approximate formula for the merger time of two stars due to the emission of gravitational waves is given by Peters & Mathews (1963):

$$t_{\text{mrg}} \approx 150 \text{ Myr} \left(\frac{M_{\odot}}{m_{\text{bh}}} \right)^3 \left(\frac{a}{R_{\odot}} \right)^4 (1 - e^2)^{7/2}. \quad (5)$$

Here e is the orbital eccentricity of the black hole binary. About 90% of the black-hole binaries formed in the cores of star clusters merge within a Hubble time due to gravitational radiation. This fraction is based on the assumption that the binary binding energies are distributed flat in $\log E_b$ between $1000 kT$ and $10000 kT$, that the eccentricities are thermal, independent of E_b (these assumptions are supported by detailed N -body simulations of smaller systems), and that the universe is 15 Gyr old (Jha et al. 1999). The specific contribution to the total merger rate of black-hole binaries from globular clusters is then about 0.04 per star cluster per million years.

4.1 Merger rate in the local universe

We estimate the number density of globular clusters in the universe to be

$$\phi_{GC} \approx 8.4 h^3 \text{ Mpc}^{-3}, \quad (6)$$

where $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Combining the specific number density of globular clusters with their contribution to the black hole merger rate results in a total rate density of black hole mergers in the universe of

$$\mathcal{R}_{GC} \approx 3.2 \times 10^{-7} h^3 \text{ yr}^{-1} \text{ Mpc}^{-3}. \quad (7)$$

We note that this figure is larger than the current best estimates of the neutron-star merger rate $\mathcal{R} \sim 2 \times 10^{-7} h^3 \text{ yr}^{-1} \text{ Mpc}^{-3}$ (Narayan et al. 1991; Phinney 1991; Portegies Zwart & Spreeuw 1996) .

4.2 LIGO observations

The current best estimate of the maximum distance within which LIGO-I can detect an inspiral event is

$$R_{\text{eff}} \approx 18 \text{ Mpc} \left(\frac{M_{\text{chirp}}}{M_{\odot}} \right)^{5/6} \quad (8)$$

(K. Thorne, private communication). Here, the “chirp” mass for a binary with component masses m_1 and m_2 is $M_{\text{chirp}} = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$. For neutron star inspiral, $m_1 = m_2 = 1.4 M_{\odot}$, so $M_{\text{chirp}} \approx 1.22 M_{\odot}$, $R_{\text{eff}} \approx 21 \text{ Mpc}$. For black-hole binaries with $m_1 = m_2 = m_{\text{bh}} = 10 M_{\odot}$, we find $M_{\text{chirp}} \approx 8.71 M_{\odot}$, $R_{\text{eff}} \approx 109 \text{ Mpc}$, and a LIGO-I detection rate of about $1.7 h^3$ per year. For $h \sim 0.65$ (Jha 1999), this results in about one detection event every two years. LIGO-II should become operational by 2007, and is expected to have R_{eff} about ten times greater than LIGO-I, resulting in a detection rate 1000 times higher—roughly one event per day.

5 Discussion

Black-hole binaries ejected from galactic nuclei, the most massive globular clusters (masses $\gtrsim 10^6 M_{\odot}$), and globular clusters which experience core collapse soon after formation tend to be very tightly bound, have high eccentricities, and merge within a few million years of ejection. These mergers therefore trace the formation of dense stellar systems with a delay of a few Gyr (the typical time required to form and eject binaries), making these systems unlikely candidates for LIGO detections, as the majority merged long ago. This effect may reduce the current merger rate by an order of magnitude, but more sensitive future gravitational wave detectors may be able to see some of

these early universe events. In fact, we estimate that the most massive globular clusters contribute about 90% of the total black hole merger rate. While their black-hole binaries merge promptly upon ejection, the longer relaxation times of these clusters mean that binaries tend to be ejected much later than in lower mass systems. Consequently, we have retained these binaries in our merger rate estimate.

We have assumed that the mass of a stellar black hole is $10 M_{\odot}$. Increasing this mass to $18 M_{\odot}$ decreases the expected merger rate by about 50%—higher mass black holes tend to have wider orbits. However, the larger chirp mass increases the signal to noise, and the distance to which such a merger can be observed increases by about 60%. The detection rate on Earth therefore increases by about a factor of three. For $6 M_{\odot}$ black holes, the detection rate decreases by a similar factor. For black-hole binaries with component masses $\gtrsim 12 M_{\odot}$, the first generation of detectors will be more sensitive to the merger itself than to the inspiral phase that precedes it (Flanagan & Hughes 1998). Since the strongest signal is expected from black-hole binaries with high-mass components, it is critically important to improve our understanding of the merger waveform. Even for lower-mass black holes (with $m_{bh} \gtrsim 10 M_{\odot}$), the inspiral signal comes from an epoch when the holes are so close together that the post-Newtonian expansions used to calculate the wave forms are unreliable. The wave forms of this “intermediate binary black hole regime” (Brady et al. 1998) are only now beginning to be explored. Finally we stress that the black-hole binaries are highly eccentric, which affects their gravitational wave signals and also influences their detectability.

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